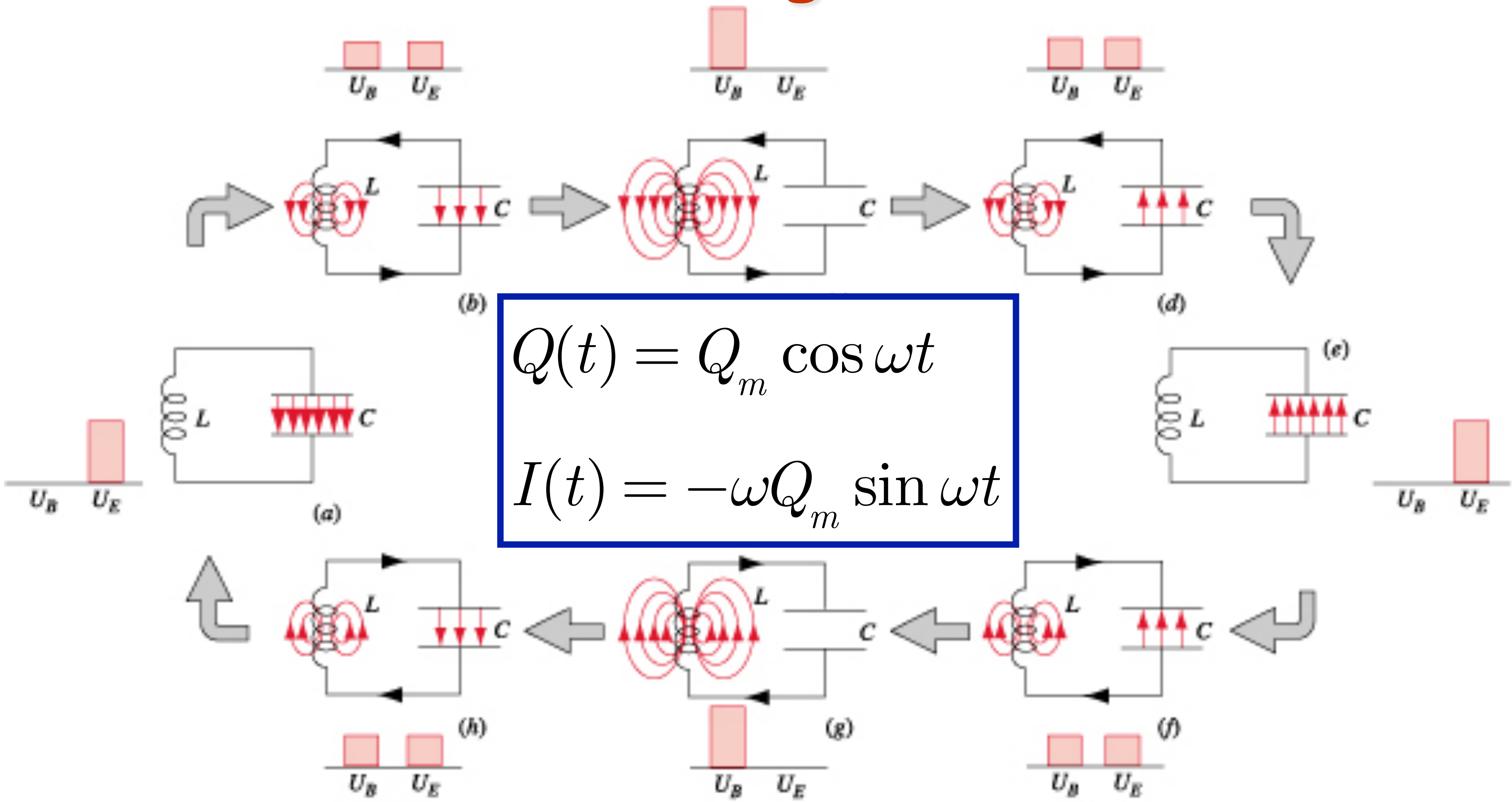
Chapter 28: Alternating-Current Circuits
Thursday November 3rd

- Review of energy and oscillations in *LC* circuits
- Alternating current theory
  - Def^n of terms, e.g., rms values
  - Resistance
  - Capacitive reactance
  - Inductive reactance
- Putting it all together – *LRC* circuits
  - Voltage/phase relations
  - Impedance
  - Resonance

Reading: up to page 501 in the text book (Ch. 28)

Ch. 28: Electromagnetic oscillations



$$Q(t) = Q_m \cos \omega t$$

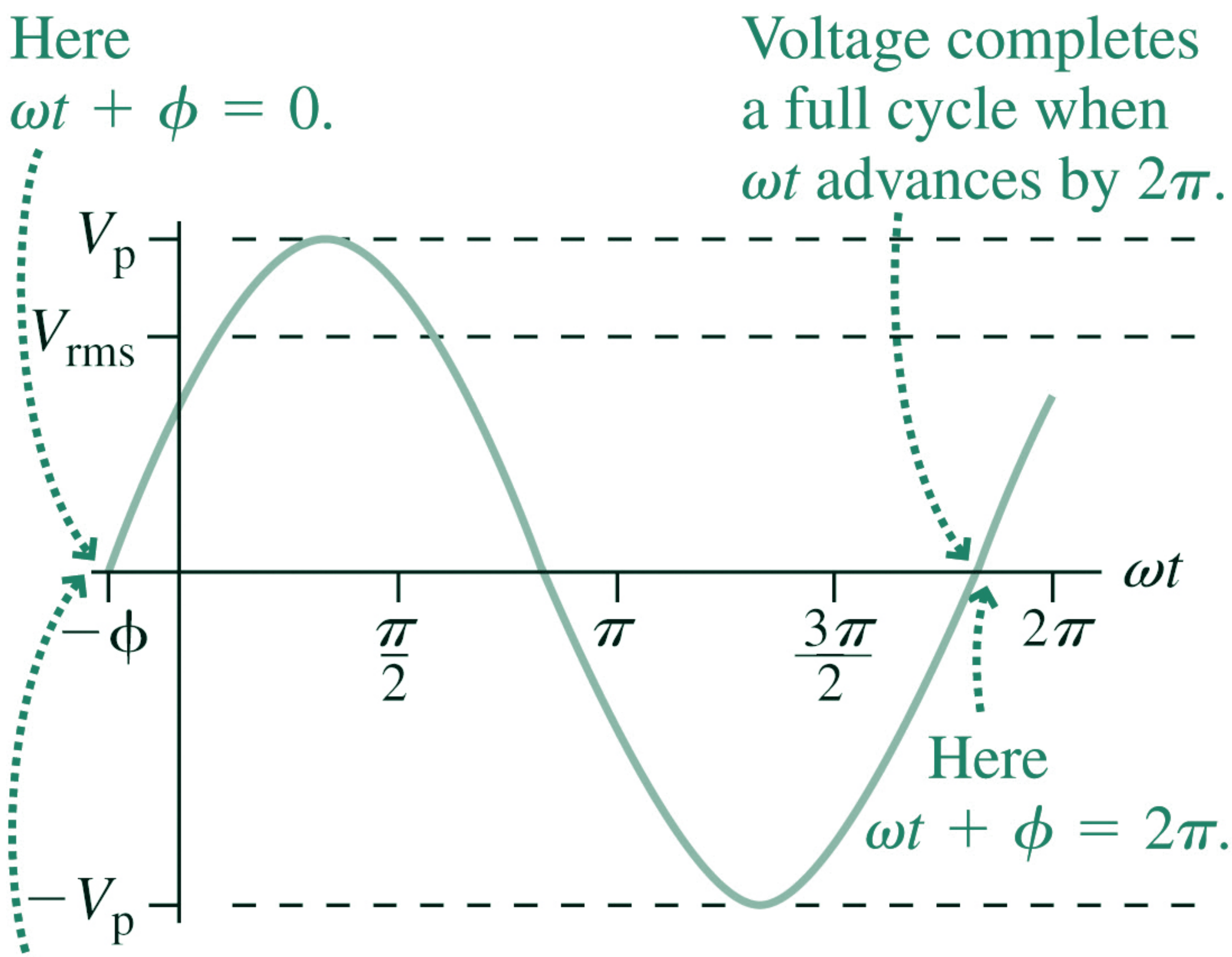
$$I(t) = -\omega Q_m \sin \omega t$$

$$U = U_B + U_E = \frac{1}{2} L I^2 + \frac{1}{2} \frac{Q^2}{C}$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Ch. 28: Alternating Current

$$V(t) = V_P \sin\left(\omega t + \phi_V\right); \qquad I(t) = I_P \sin\left(\omega t + \phi_I\right)$$



Sine curve starts at $\omega t = -\pi/6$ or -30^{o}

Angular frequency:

$$\omega = 2\pi f$$

In this example:

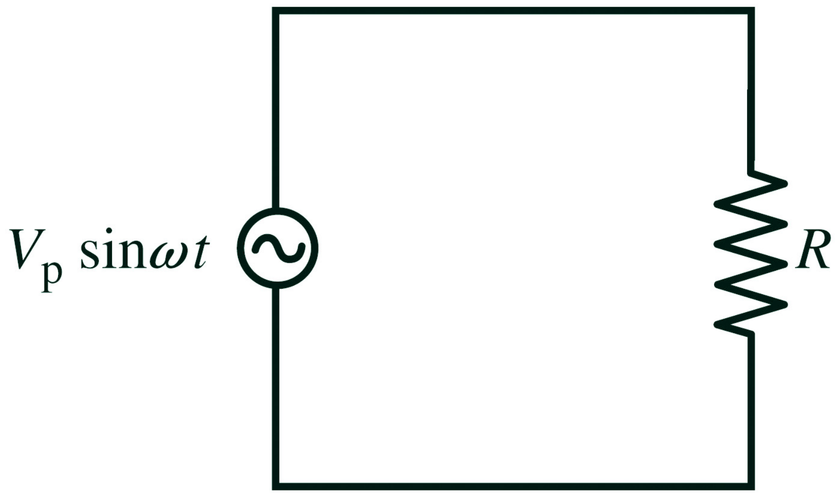
$\phi_V = +\pi/6$ or 30^{o}

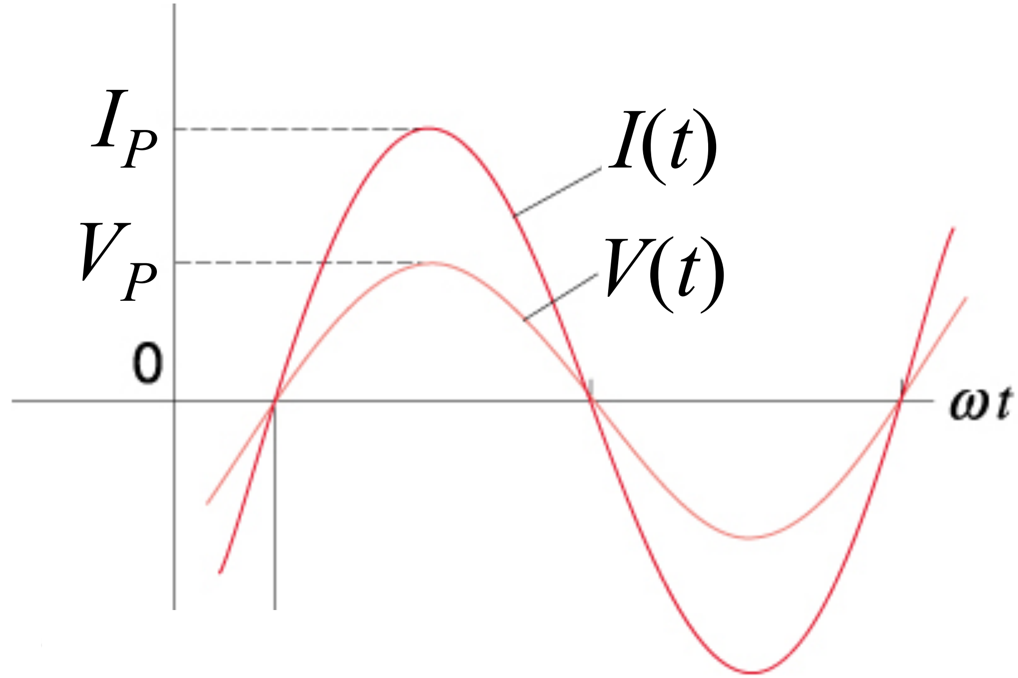
Root-Mean-Square:

$$V_{rms} = \frac{V_P}{\sqrt{2}}$$

$$I_{rms} = \frac{I_P}{\sqrt{2}}$$

AC circuits: the resistive term





Voltage (driving term):

$$V(t) = V_P \sin \omega t$$

Current response:

$$I(t) = \frac{V}{R} = \frac{V_P}{R} \sin \omega t$$

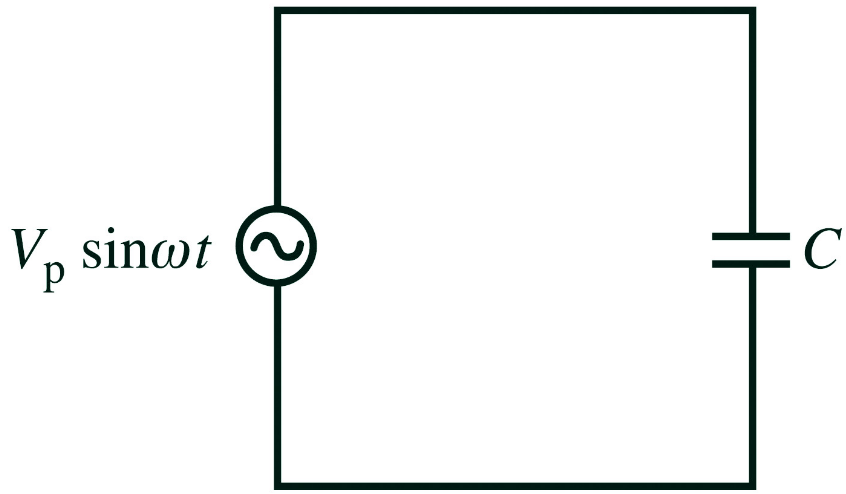
For a resistor **ONLY**:

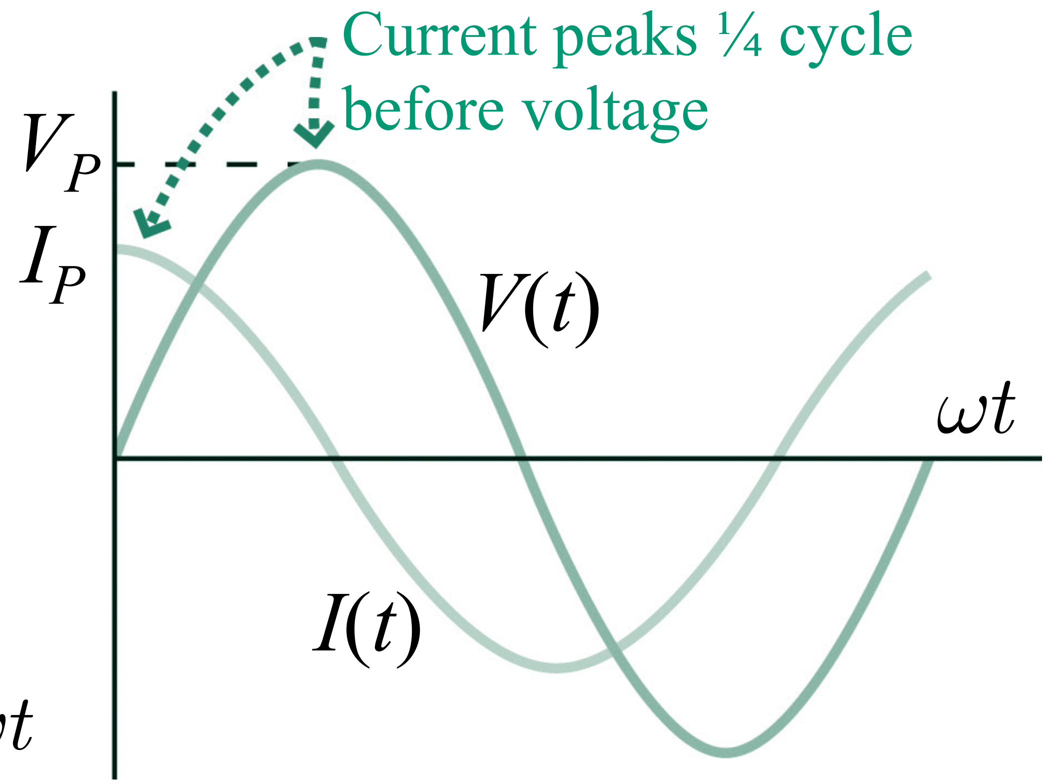
Current and voltage in phase

$$\Rightarrow I_P = V_P / R$$

$$I_{rms} = V_{rms} / R$$

AC circuits: the capacitive term





Charge (driving term):

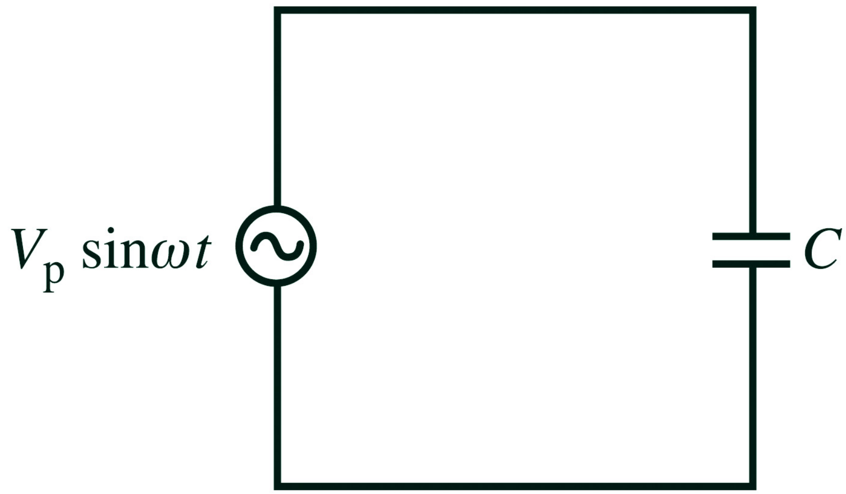
$$Q(t) = CV(t) = CV_P \sin \omega t$$

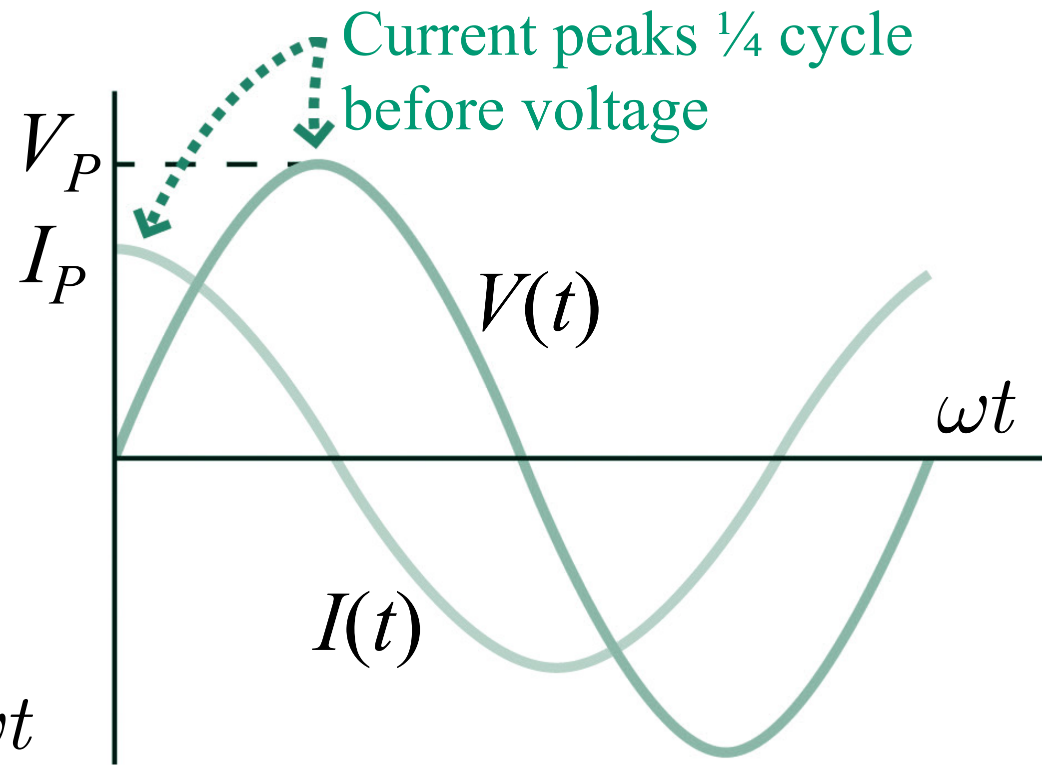
Current response:

$$I(t) = \frac{dQ}{dt} = CV_P \frac{d}{dt}(\sin \omega t)$$

$$= \omega CV_P \cos \omega t = \omega CV_P \sin(\omega t + \tfrac{\pi}{2})$$

AC circuits: the capacitive term





Charge (driving term):

$$Q(t) = CV(t) = CV_P \sin \omega t$$

Current response:

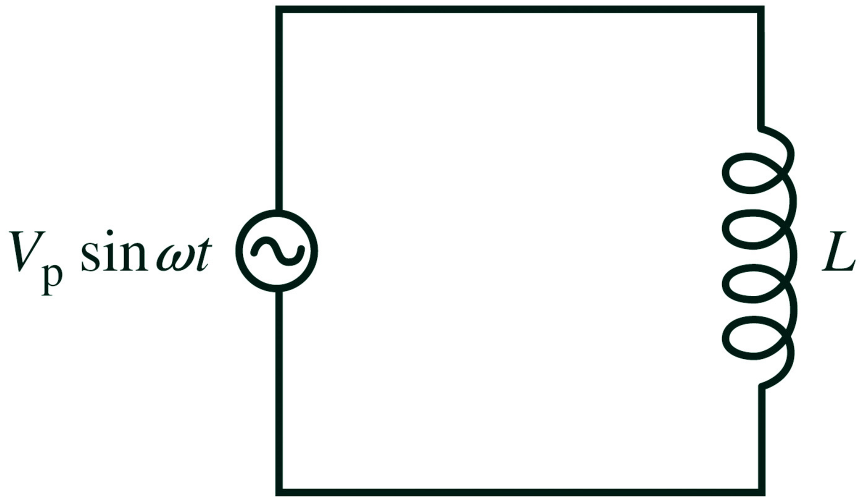
$$I(t) = \omega C V_P \sin(\omega t + \tfrac{\pi}{2})$$

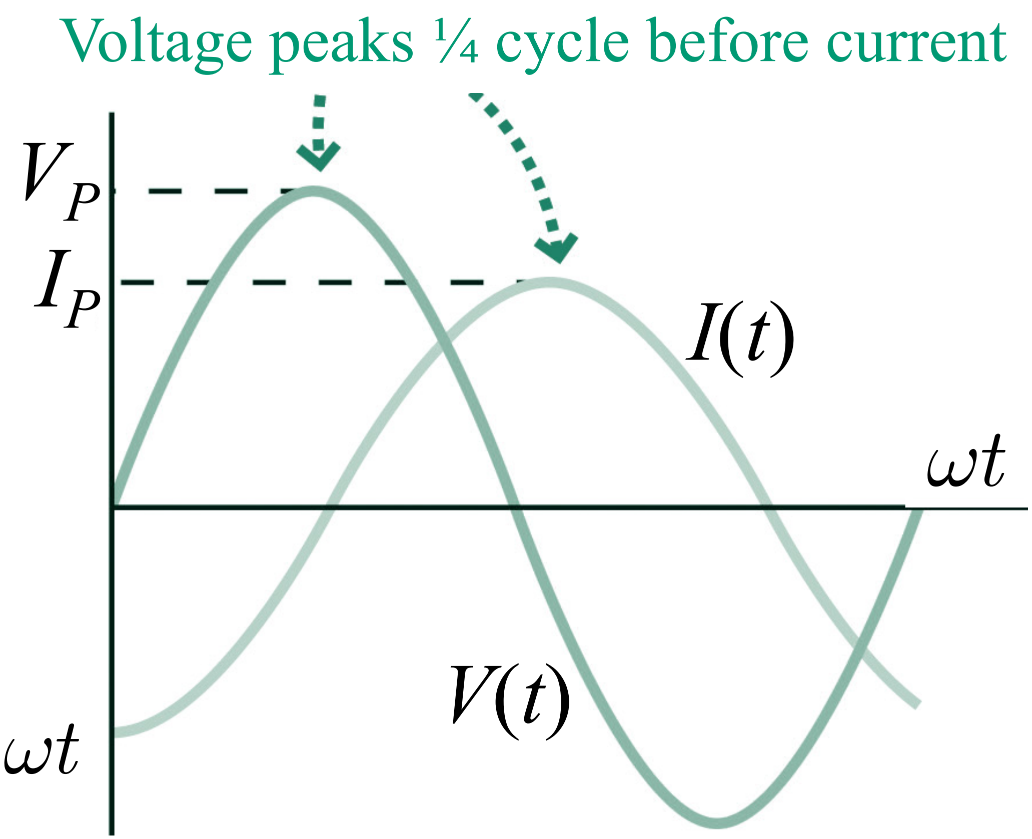
Current leads voltage by 90°

$$I_P = \frac{V_P}{1/\omega C} = \frac{V_P}{X_C}$$

Capacitive reactance: $X_C = 1/\omega C$ (units - Ω)

AC circuits: the inductive term





Current (driving term):

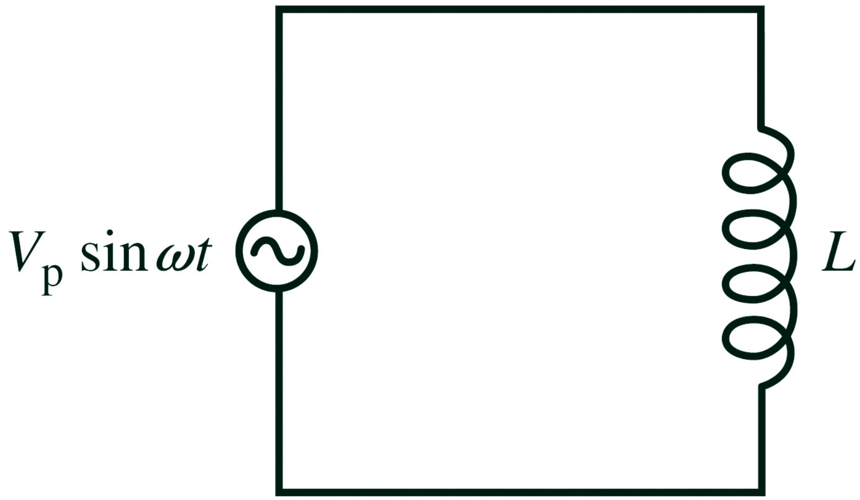
$$V_L = -L dI / dt = -V_P \sin\omega t$$

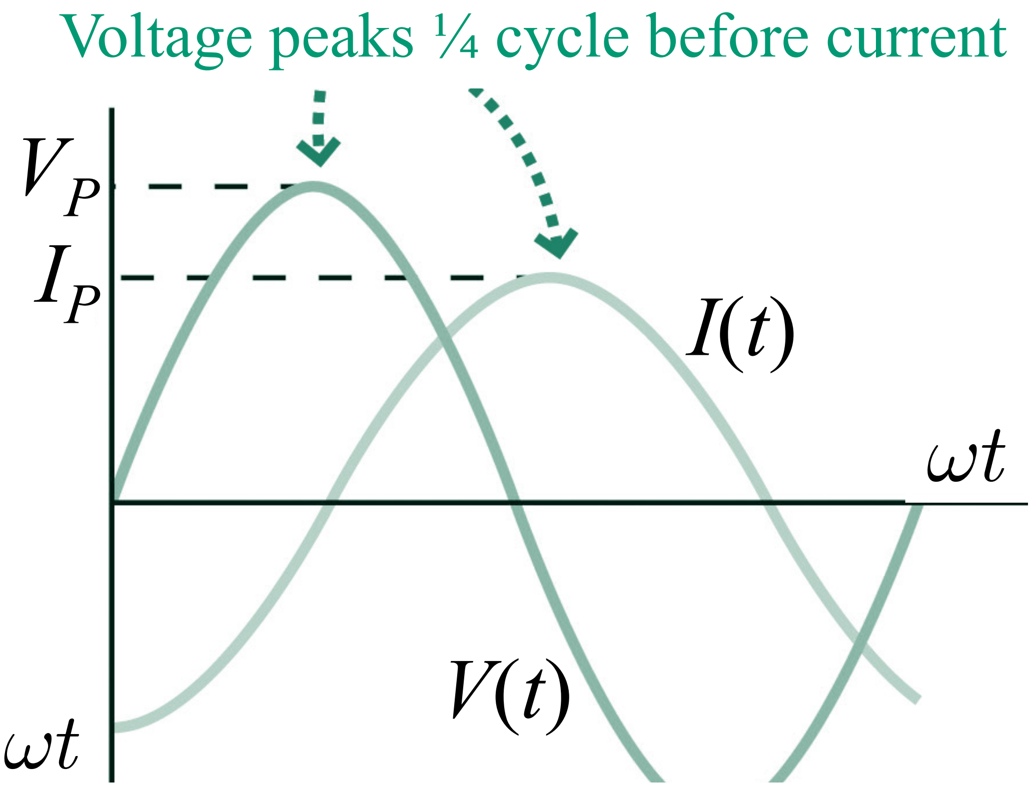
Current response:

$$I(t) = \frac{V_P}{L}\int \sin\omega t \; dt$$

$$= -\frac{V_P}{\omega L}\cos\omega t = \frac{V_P}{\omega L}\sin(\omega t - \tfrac{\pi}{2})$$

AC circuits: the inductive term





Current (driving term):

$$V_L = -L\,dI\,/\,dt = -V_P \sin\omega t$$

Voltage leads current by 90°

Current response:

$$I(t) = \frac{V_P}{\omega L}\sin(\omega t - \tfrac{\pi}{2})$$

$$I_P = \frac{V_P}{\omega L} = \frac{V_P}{X_L}$$

Inductive reactance: $X_L = \omega L$ (units - Ω)

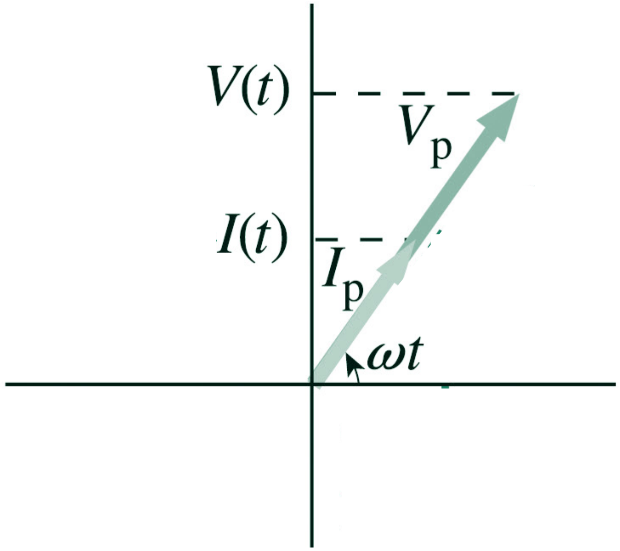
Summary

Table 28.1 Amplitude and Phase Relations in Circuit Elements

Circuit Element	Peak Current versus Voltage	Phase Relation
Resistor	$I_p = \dfrac{V_p}{R}$	V and I in phase
Capacitor	$I_p = \dfrac{V_p}{X_C} = \dfrac{V_p}{1/\omega C}$	I leads V by 90°
Inductor	$I_p = \dfrac{V_p}{X_L} = \dfrac{V_p}{\omega L}$	V leads I by 90°

Phasor Diagrams

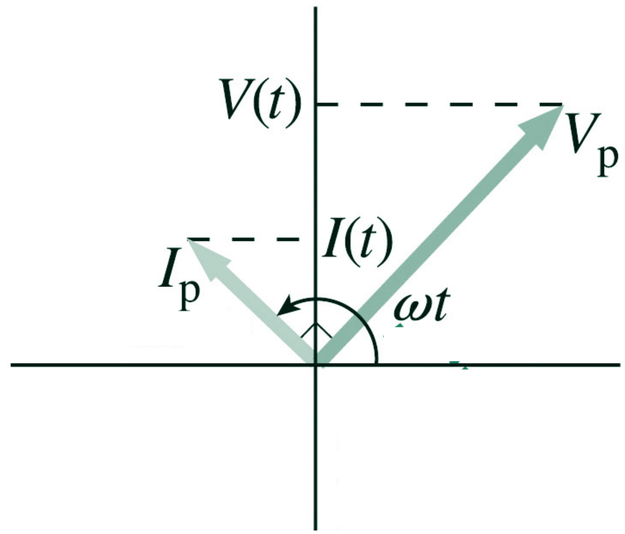
Resistor



$$V_{Rp} = I_p R$$

V, *I* in phase

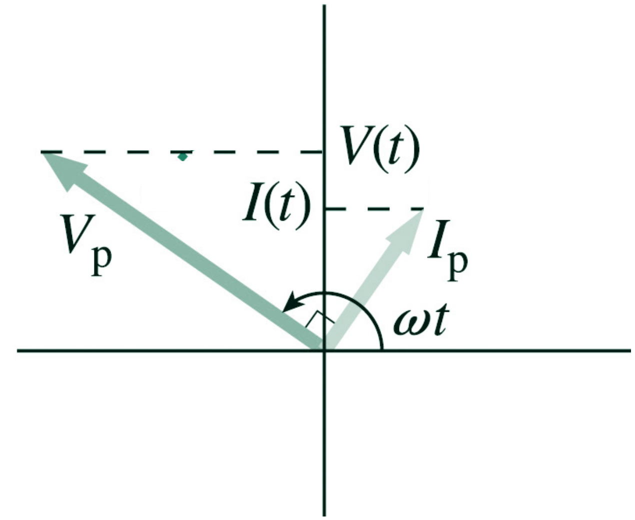
Capacitor



$$V_{Cp} = I_p X_C$$

V lags *I* by 90°

Inductor



$$V_{Lp} = I_p X_L$$

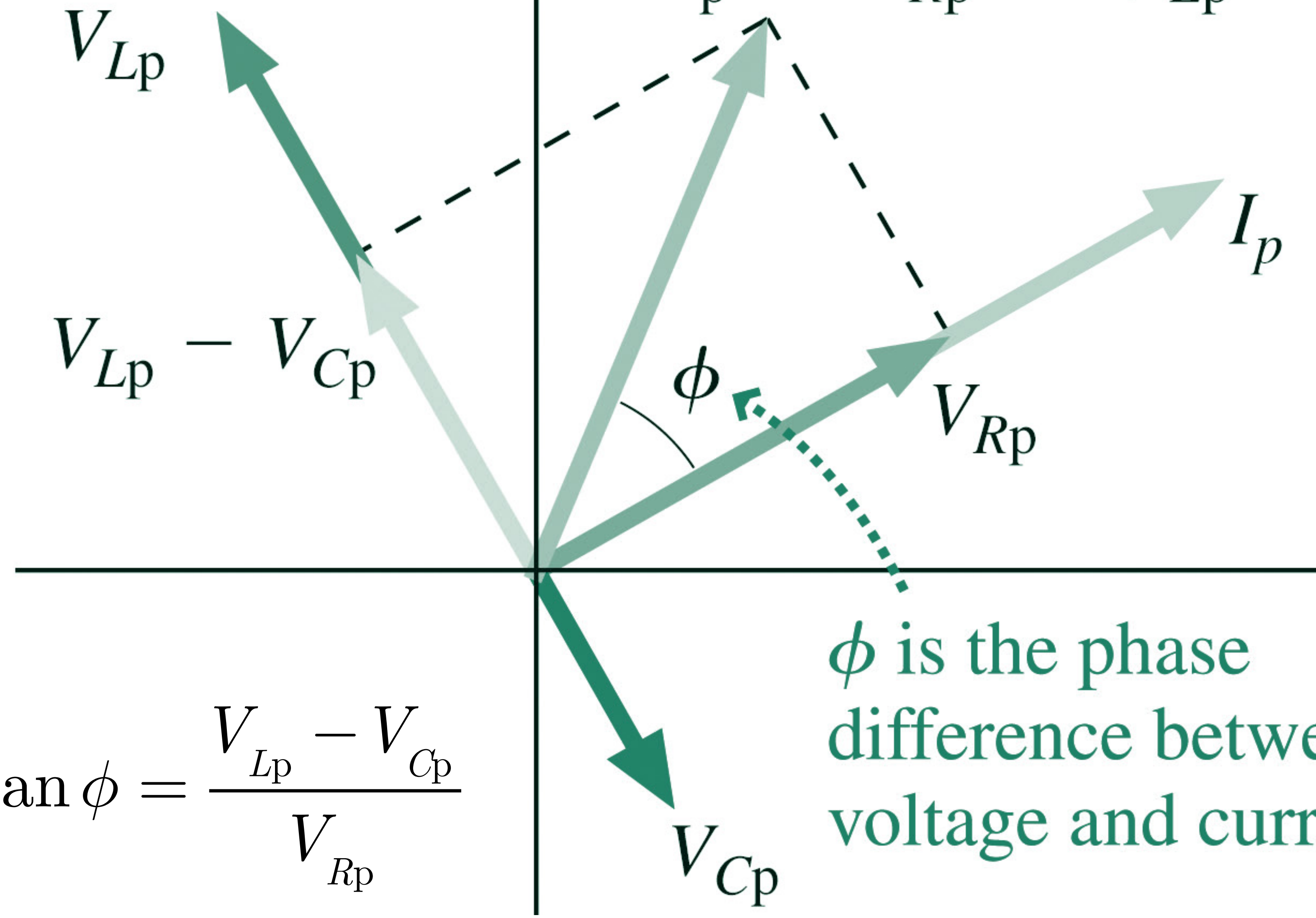
V leads *I* by 90°

$$V(t) = \sin \omega t; \qquad I(t) = \sin(\omega t - \phi)$$

http://en.wikipedia.org/wiki/Phasor_(sine_waves)

Phasor Diagrams: Adding the Voltages

$$V_p = \sqrt{V_{Rp}^{\,2} + (V_{Lp} - V_{Cp})^2}$$



$$\tan\phi = \frac{V_{Lp} - V_{Cp}}{V_{Rp}}$$

Phasor Diagrams: Adding the Voltages

$$V_p = \sqrt{I_p R^2 + \left(I_p X_L - I_p X_C\right)^2}$$

Modified Ohm's law:

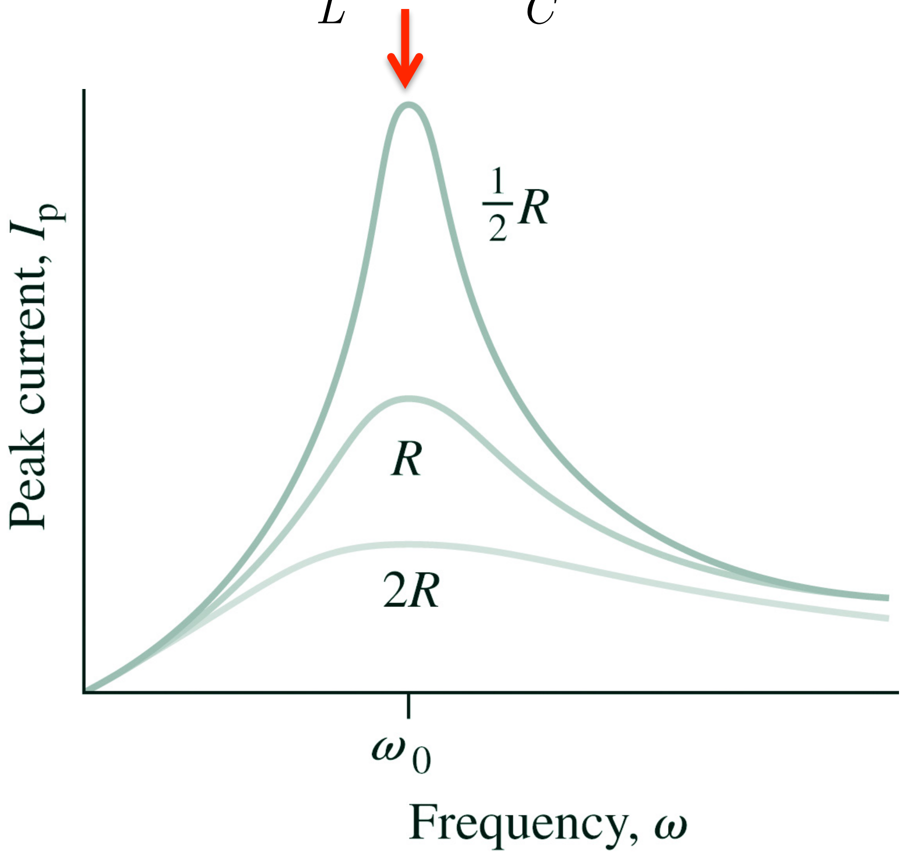
$$\Rightarrow I_p = \frac{V_p}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{V_p}{Z}$$

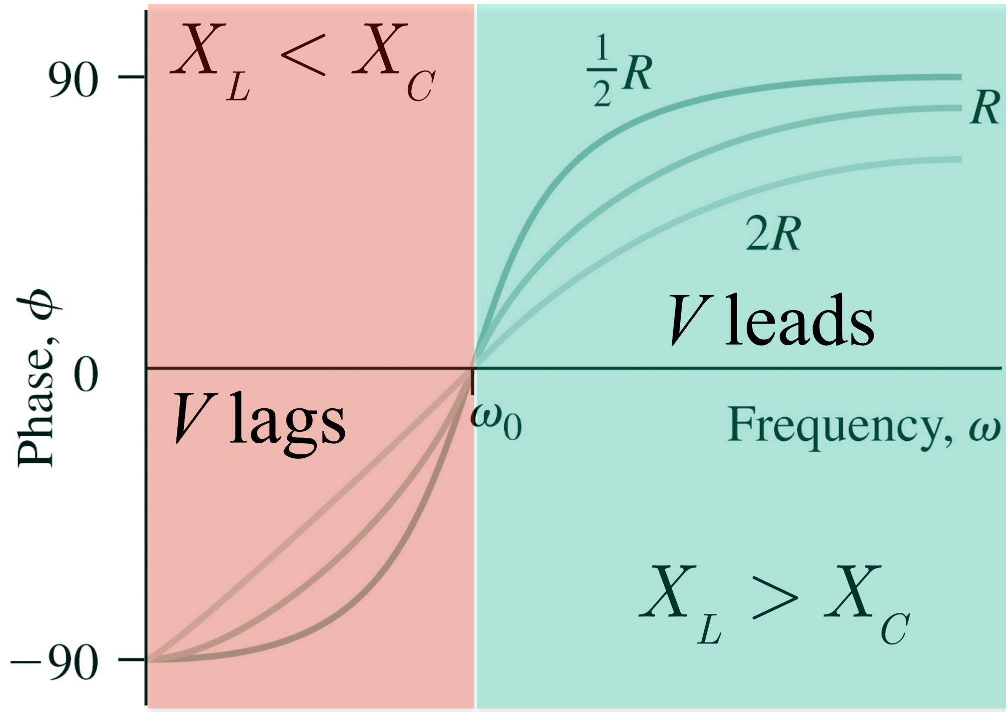
Impedance: $$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$ $\left[\begin{array}{l}\text{Units:}\\ \text{ohms}\end{array}\right]$

Phase: $$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

Resonance

$X_L = X_C$



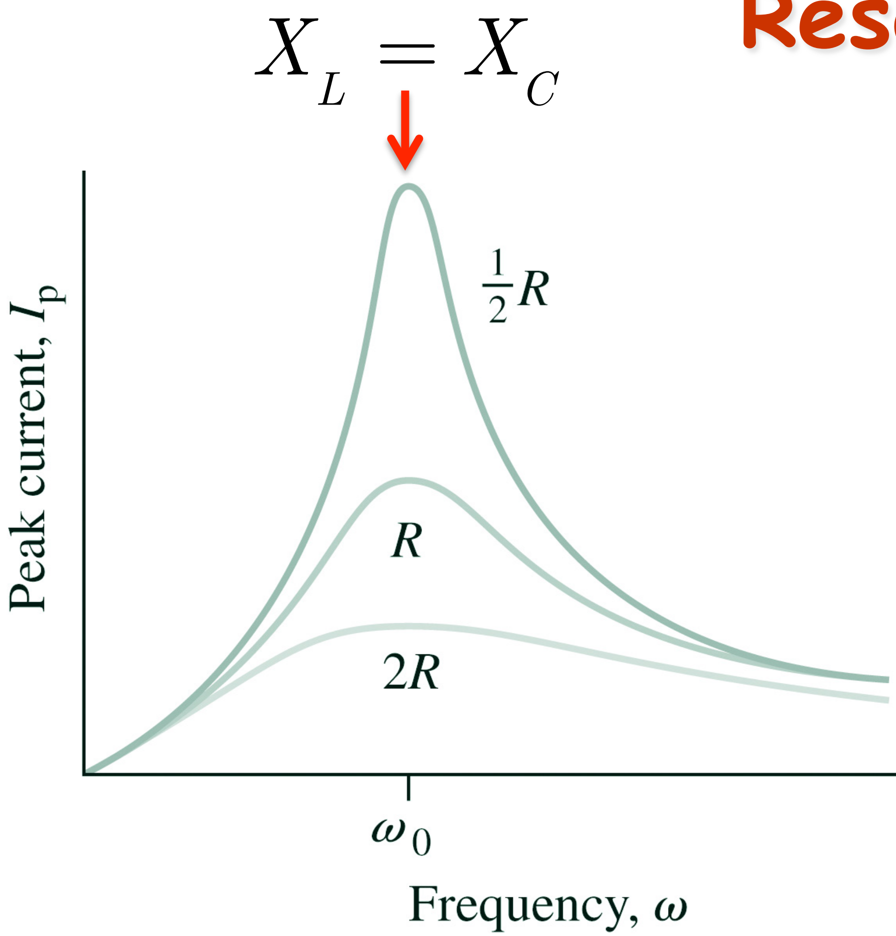


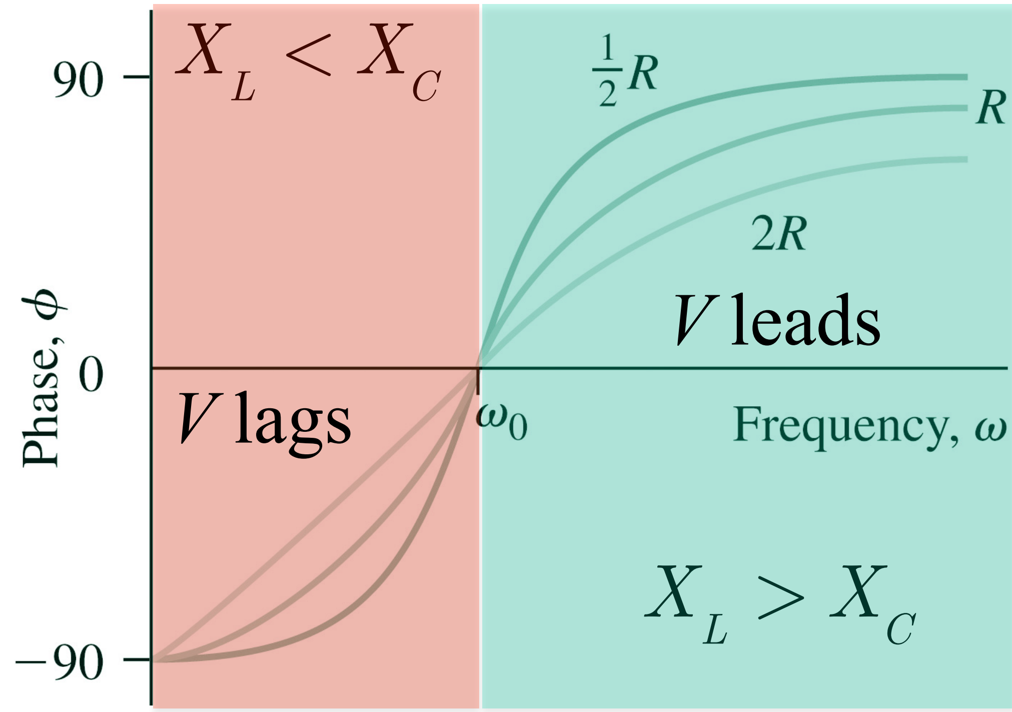
$$I_p = \frac{V_p}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

$$\omega_o = \sqrt{\frac{1}{LC}}$$

At resonance, $Z = R$, and $\phi = 0$ (just like a DC circuit)

Resonance





Power delivered to the circuit:

$$\langle P \rangle = \tfrac{1}{2} I_{\mathrm{p}} V_{\mathrm{p}} \cos\phi = I_{rms} V_{rms} \cos\phi$$